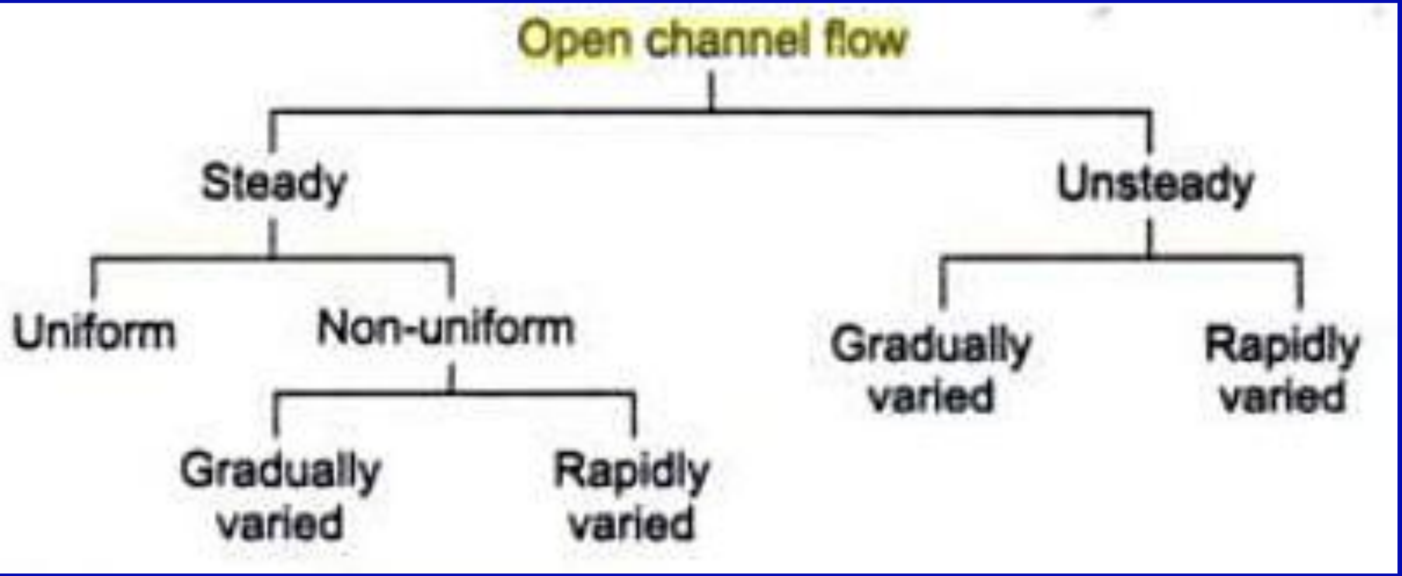
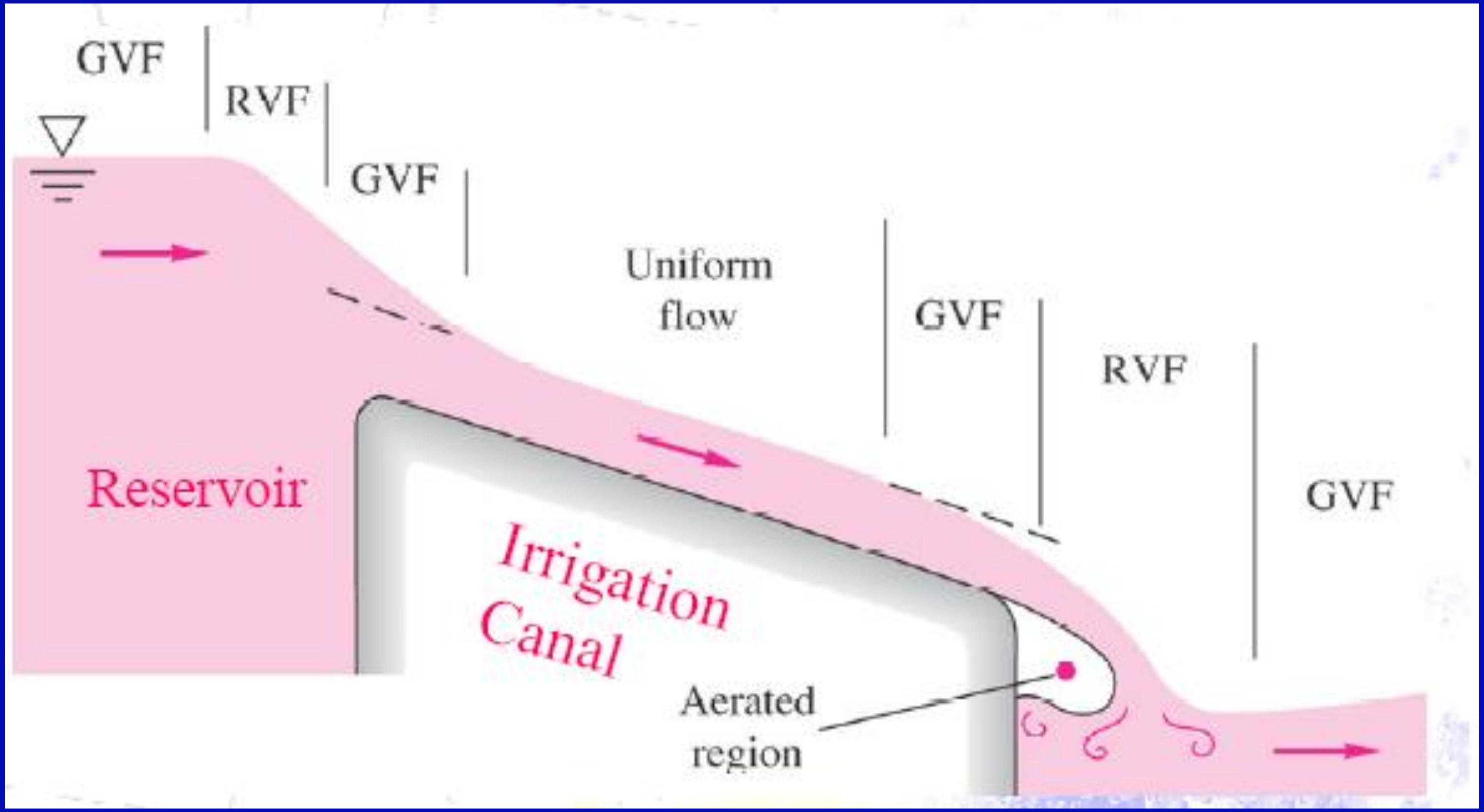
GRADUALLY VARIED FLOW

FLOW CLASSIFICATION



- **Uniform (normal) flow: Depth is constant at every section along length of channel**
- **Non-uniform (varied) flow: Depth changes along channel**
  - Rapidly-varied flow: Depth changes suddenly
  - Gradually-varied flow: Depth changes gradually

FLOW CLASSIFICATION



- RVF: Rapidly-varied flow
- GVF: Gradually-varied flow

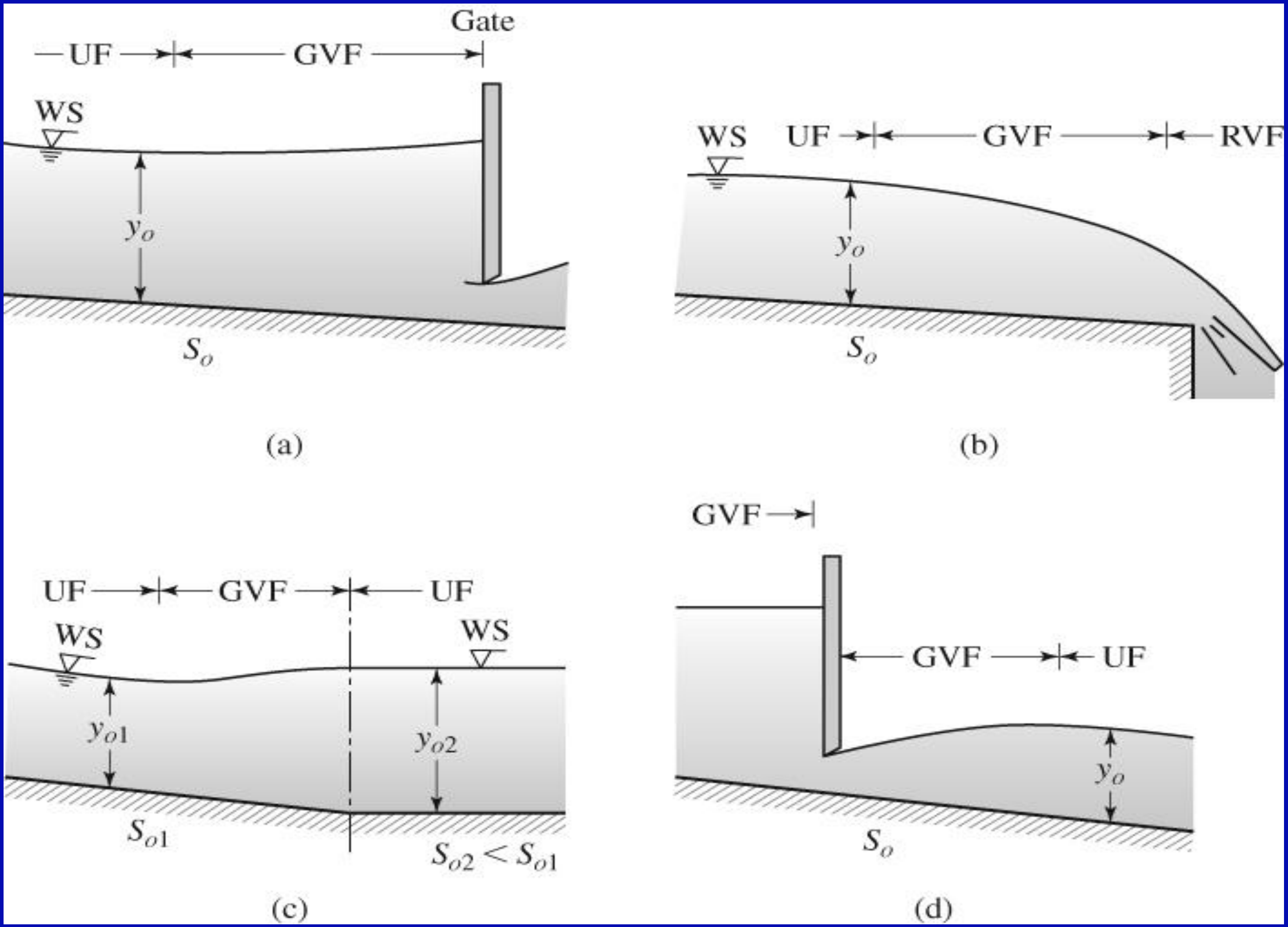


Figure 14.1 Examples for gradually varied flow in open channels.

ASSUMPTIONS FOR GRADUALLY-VARIED FLOW

1. The channel is prismatic and the flow is steady.

2. The bed slope, So, is relatively small.

3. The velocity distribution in the vertical section is uniform and the kinetic energy correction factor is close to unity.

4. Streamlines are parallel and the pressure distribution is hydrostatic.

5. The channel roughness is constant along its length and does not depend on the depth of flow.

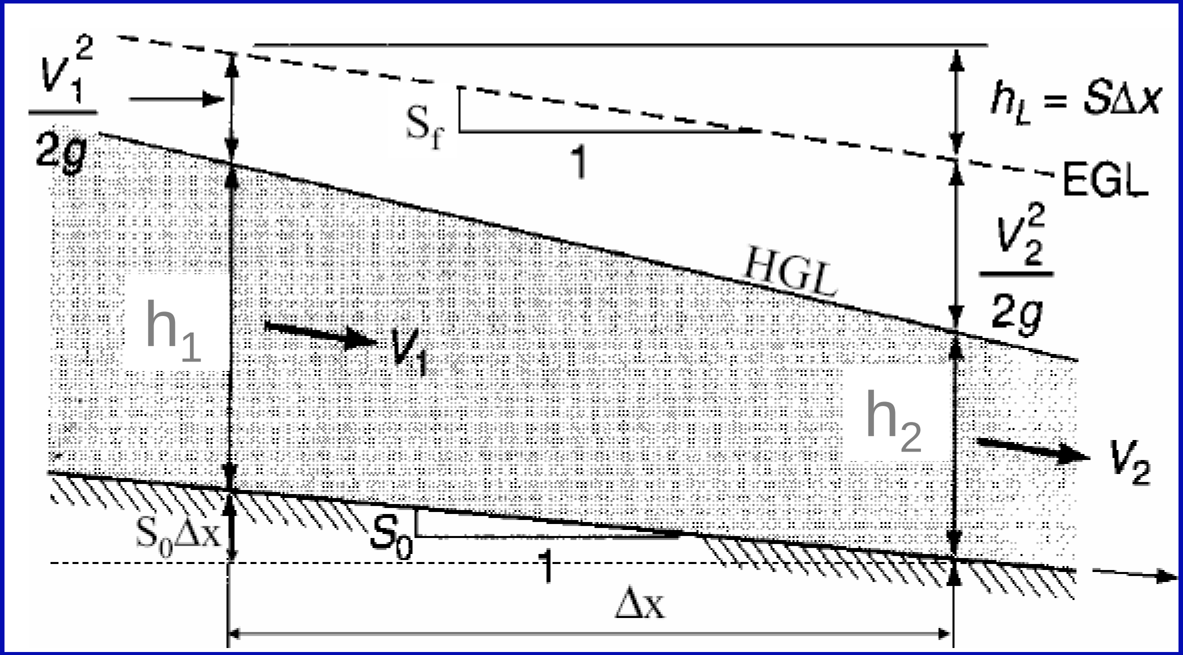
ANALYSIS OF GRADUALLY-VARIED FLOW

- Characteristics of gradually varied flow
  - Water depth and velocity change gradually,
  - Flow is nonuniform,
  - water surface changes smoothly and continuously,
  - Friction loss along the channel is not negligible.

- Tasks
  - Deduce the trend of water surface change (classification of surface profiles)
  - Calculate water levels and velocity along the course of the channel (quantitative evaluation)

- Analysis method
  - Bernoulli equation
  - Continuity (mass conservation) equation

THE EQUATIONS FOR GRADUALLY VARIED FLOW



$$H = \frac{V^2}{2g} + h + z$$

$$\frac{dH}{dx} = \frac{d}{dx}\left(\frac{V^2}{2g}\right) + \frac{dh}{dx} + \frac{dz}{dx}$$

$$dH/dx = -S,\ dz/dx = -S_0$$

$S=S_f$: slope of EGL

THE EQUATIONS FOR GRADUALLY VARIED FLOW

It should be noted that *the slope is defined as the sine of the slope angle* and that is assumed positive if it descends in the direction of flow and negative if it ascends. Hence,

$$-dH / dx = S, \; -dz / dx = S_0$$

It should be noted that the friction loss dh is always a negative quantity in the direction of flow (unless outside energy is added to the course of the flow) and that the change in the bottom elevation dz is a negative quantity when the slope descends.

In the other words, they are negative because H and z decrease in the flow direction

THE EQUATIONS FOR GRADUALLY VARIED FLOW

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right)=\frac{d}{dx}\left(\frac{Q^2}{2gA^2}\right)\frac{dh}{dh}=\frac{dh}{dx}\frac{d}{dh}\left(\frac{Q^2}{2gA^2}\right)=-\frac{dh}{dx}\frac{Q^2}{gA^3}\frac{dA}{dh}=-\frac{dh}{dx}\frac{Q^2B}{gA^3}$$

$$-S=-\frac{dh}{dx}\frac{Q^2B}{gA^3}+\frac{dh}{dx}-S_0=\left(1-\frac{Q^2B}{gA^3}\right)\frac{dh}{dx}-S_0$$

$$\frac{dh}{dx}=\frac{S_0-S}{1-Q^2B/gA^3} \Rightarrow \frac{dh}{dx}=\frac{S_0-S}{1-Fr^2}$$

General governing Equation for GVF

If dh/dx is positive the depth is increasing otherwise decreasing

DERIVATION OF GVF EQUATION

$$\frac{dh}{dx} = \frac{S_0 - S}{1 - Fr^2}$$

For any cross-section

$$\frac{dh}{dx} = S_o \left\{ \frac{1 - \left(\frac{y_o}{y}\right)^3}{1 - \left(\frac{y_c}{y}\right)^3} \right\}$$

Wide rectangular section (Using Chezy equation for S_f)

$$C = \frac{V}{\sqrt{RS_f}}$$

$$\frac{dh}{dx} = S_o \left\{ \frac{1 - \left(\frac{y_o}{y}\right)^{10/3}}{1 - \left(\frac{y_c}{y}\right)^3} \right\}$$

Wide rectangular section (Using Manning's formula for S_f)

WATER SURFACE PROFILES

For a given channel with a known Q = Discharge, n = Manning coefficient, and S_o = channel bed slope, y_c = critical water depth and y_o = uniform flow depth can be computed.

There are three possible relations between y_o and y_c as

1) $y_o > y_c$,

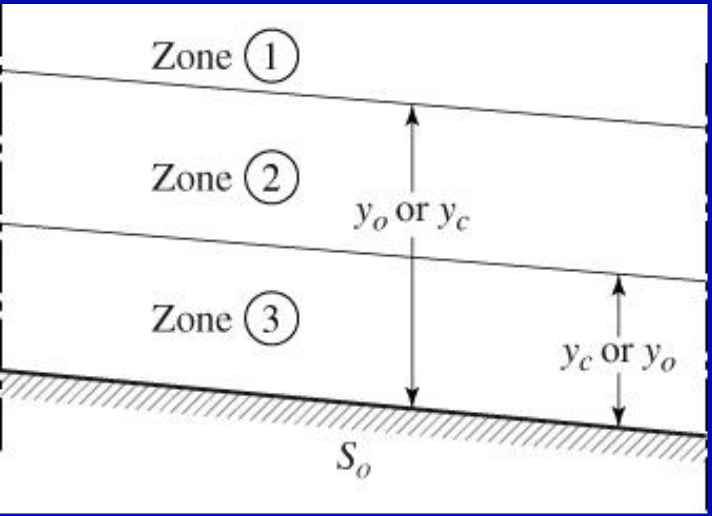
2) $y_o < y_c$,

3) $y_o = y_c$.

WATER SURFACE PROFILES CLASSIFICATION

For each of the five categories of channels (in previous slide), lines representing the critical depth (y_c) and normal depth (y_o) (if it exists) can be drawn in the longitudinal section.

These would divide the whole flow space into three regions as: (y: non-uniform depth)



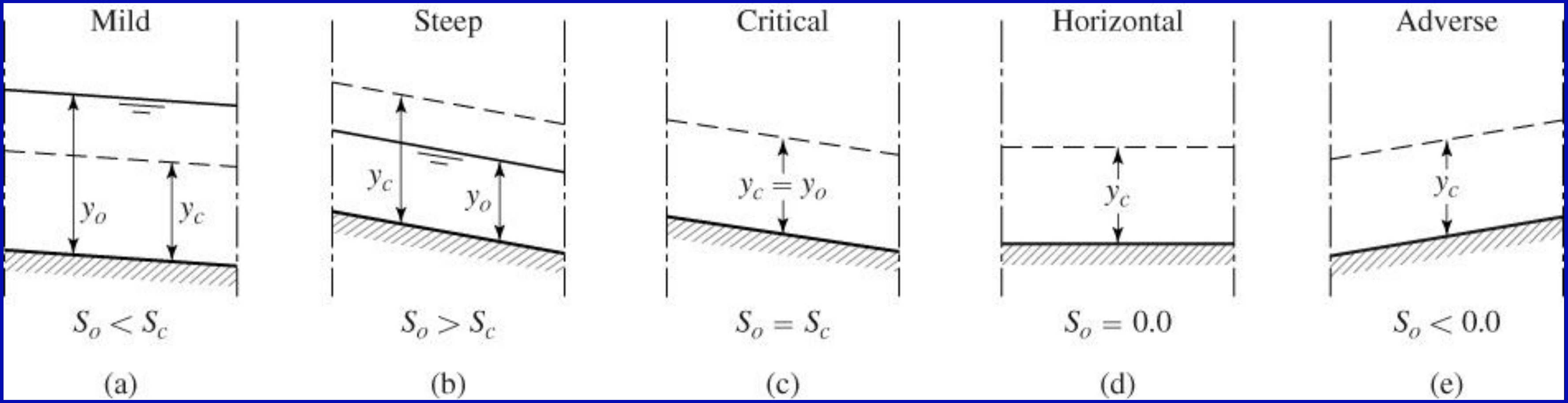
Zone 1: Space above the topmost line,
$y > y_o > y_c$, $y > y_c > y_o$

Zone 2: Space between top line and the next lower line
$y_o > y > y_c$, $y_c > y > y_o$

Zone 3: Space between the second line and the bed.
$y_o > y_c > y$, $y_c > y_o > y$

WATER SURFACE PROFILES CLASSIFICATION

Number	Channel category	Symbol	Characteristic condition	Remark
1	Mild slope	M	$y_0 > y_c$	Subcritical flow at normal depth
2	Steep slope	S	$y_c > y_0$	Supercritical flow at normal depth
3	Critical slope	C	$y_c = y_0$	Critical flow at normal depth
4	Horizontal bed	H	$S_0 = 0$	Cannot sustain uniform flow
5	Adverse slope	A	$S_0 < 0$	Cannot sustain uniform flow



WATER SURFACE PROFILES CLASSIFICATION

For the horizontal ($S_o = 0$) and adverse slope ($S_o < 0$) channels,

$$Q = \frac{1}{n} A R^{2/3} S_o^{1/2}$$

Horizontal channel: $S_o = 0 \rightarrow Q = 0$

Adverse channel: $S_o < 0$ Q cannot be computed,

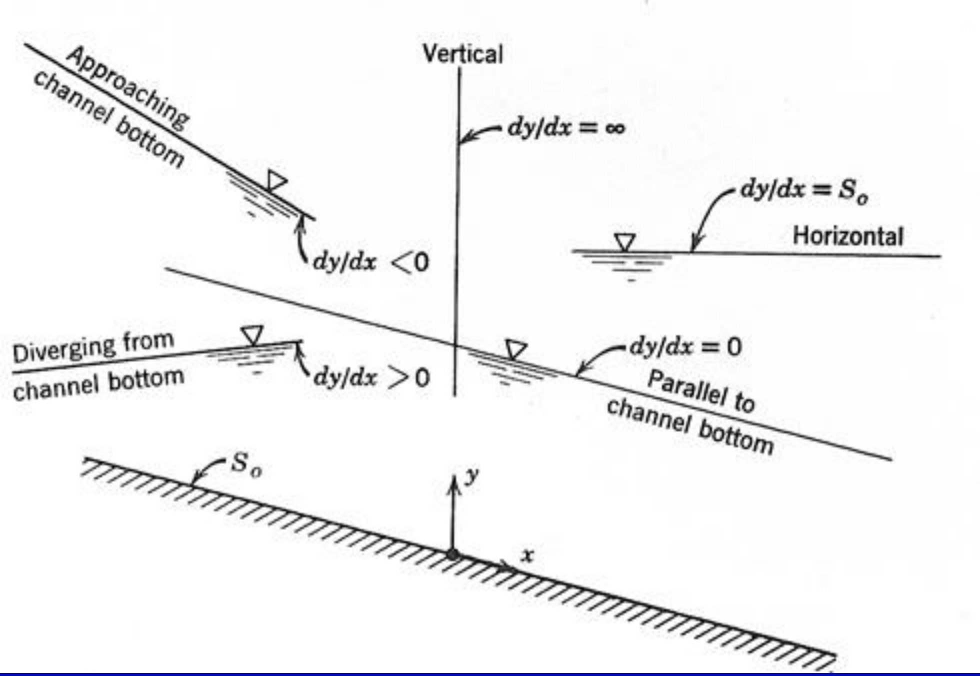
For the horizontal and adverse slope channels,
***the uniform flow depth y_o does not exist*.**

WATER SURFACE PROFILES CLASSIFICATION

For a given Q, n, and S_o at a channel,

y_o = Uniform flow depth, y_c = Critical flow depth, y = Non-uniform flow depth.

The depth y is measured vertically from the channel bottom, the slope of the water surface dy / dx is relative to this channel bottom.



the prediction of surface profiles from the analysis of

$$\frac{dh}{dx} = \frac{S_0 - S}{1 - Fr^2}$$

Classification of Profiles According to dy/dl

dl=dx

1) $dy/dx>0$; the depth of flow is increasing with the distance. (A rising Curve)

2) $dy/dx<0$; the depth of flow is decreasing with the distance. (A falling Curve)

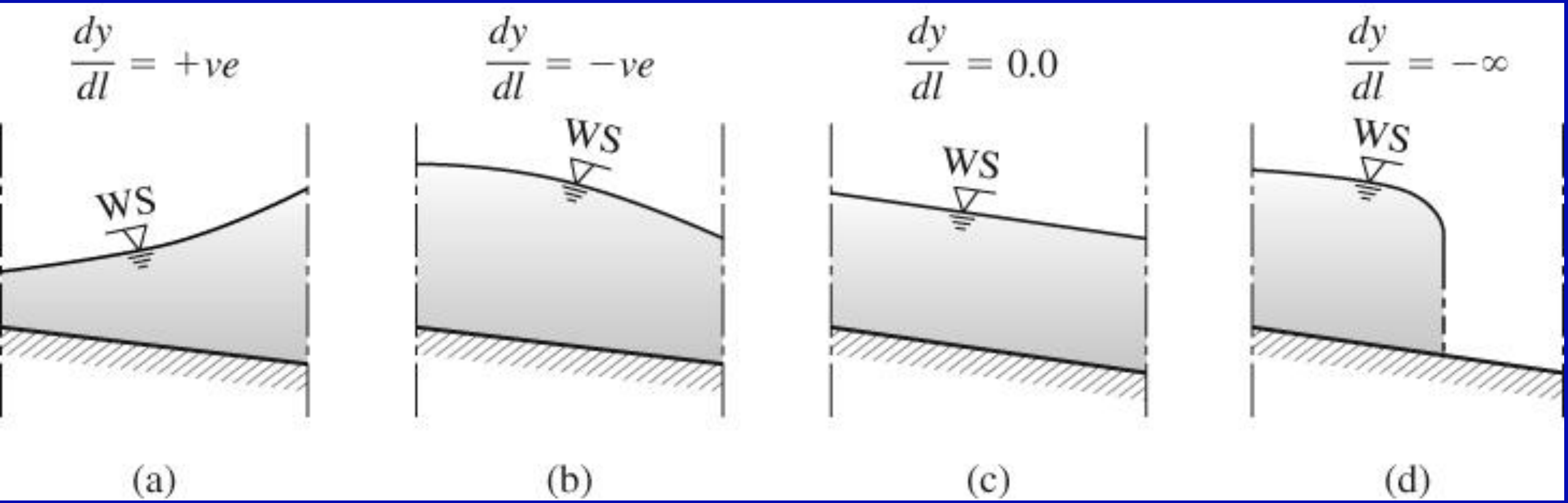
3) $dy/dx=0$. The flow is uniform Sf=So

4) $dy/dx = -\infty$. The water surface forms a right angle with the channel bed.

5) $dy/dx=\infty/\infty$. The depth of flow approaches a zero.

6) $dy/dx= S_o$ The water surface profile forms a horizontal line. This is special case of the rising water profile

WATER SURFACE PROFILES CLASSIFICATION



Classification of profiles according to *dy / dl or (dh/dx)*.

$$\frac{dh}{dx} = \frac{S_0 - S}{1 - Fr^2}$$

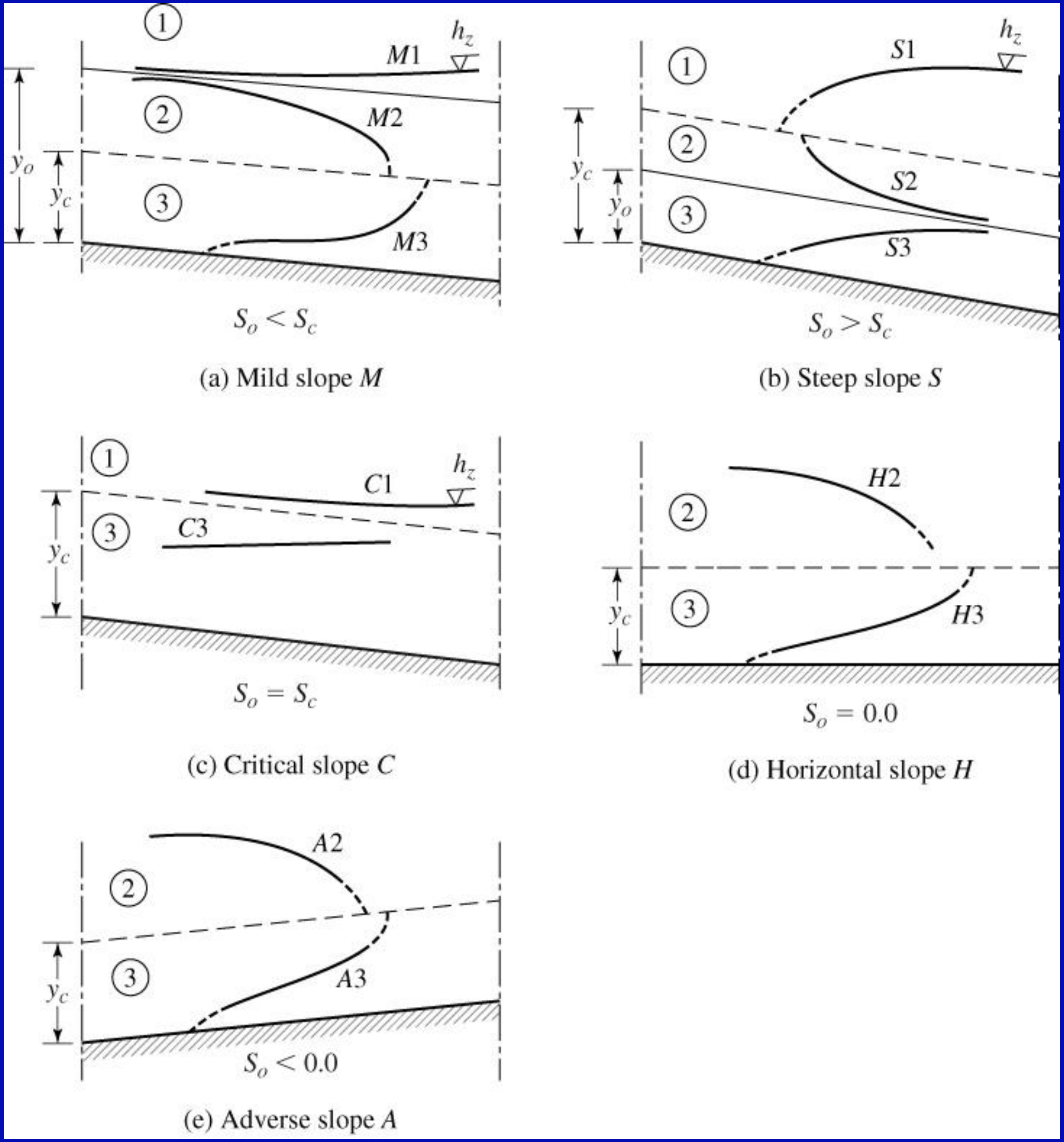
GRAPHICAL REPRESENTATION OF THE GVF

Zone 1: $y > y_o > y_c$

Zone 2: $y_o > y > y_c$

Zone 3: $y_o > y_c > y$

$$\frac{dh}{dx} = S_o \left\{ \frac{1 - \left(\frac{y_o}{y}\right)^3}{1 - \left(\frac{y_c}{y}\right)^3} \right\}$$



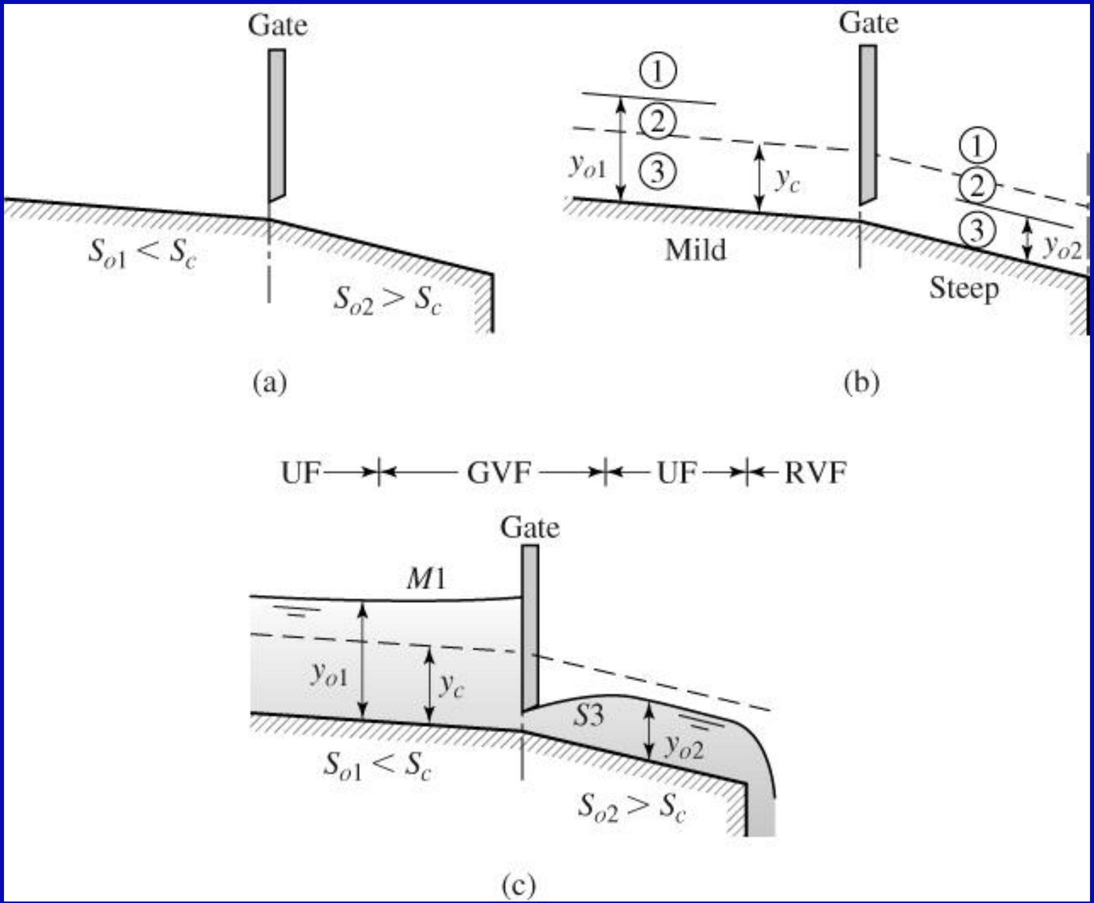
Outlining Water Surface Profiles

1. Determine the type of bed slope (mild, steep, critical, horizontal, or adverse) in each reach of the channel according to the bed slope as compared to the critical slope. One can also compare the normal depth with the critical depth if the bed slope is not given.
2. Plot the critical depth which is constant along the entire channel. It does not depend on the bed slope. Also plot the normal depths in the different reaches away from any hydraulic structures and/or points of variation in bed slope, as shown in Figure 14.7a. Water surface profiles should bridge these normal depths.
3. According to the type of bed, select the appropriate curves from the corresponding profiles shown in Figure 14.6. If the water depth needs to increase above the critical depth, then consider the corresponding curve from Zone 1. If this increase is encountered below the critical depth, then pick the corresponding curve from Zone 3. If the depth needs to decrease, one should always select the corresponding curve from Zone 2, as shown in Figure 14.7b.

Please read your text book for the rest. Page 451

Example

Draw water surface profile for two reaches of the open channel given in Figure below. A gate is located between the two reaches and the second reach ends with a sudden fall.



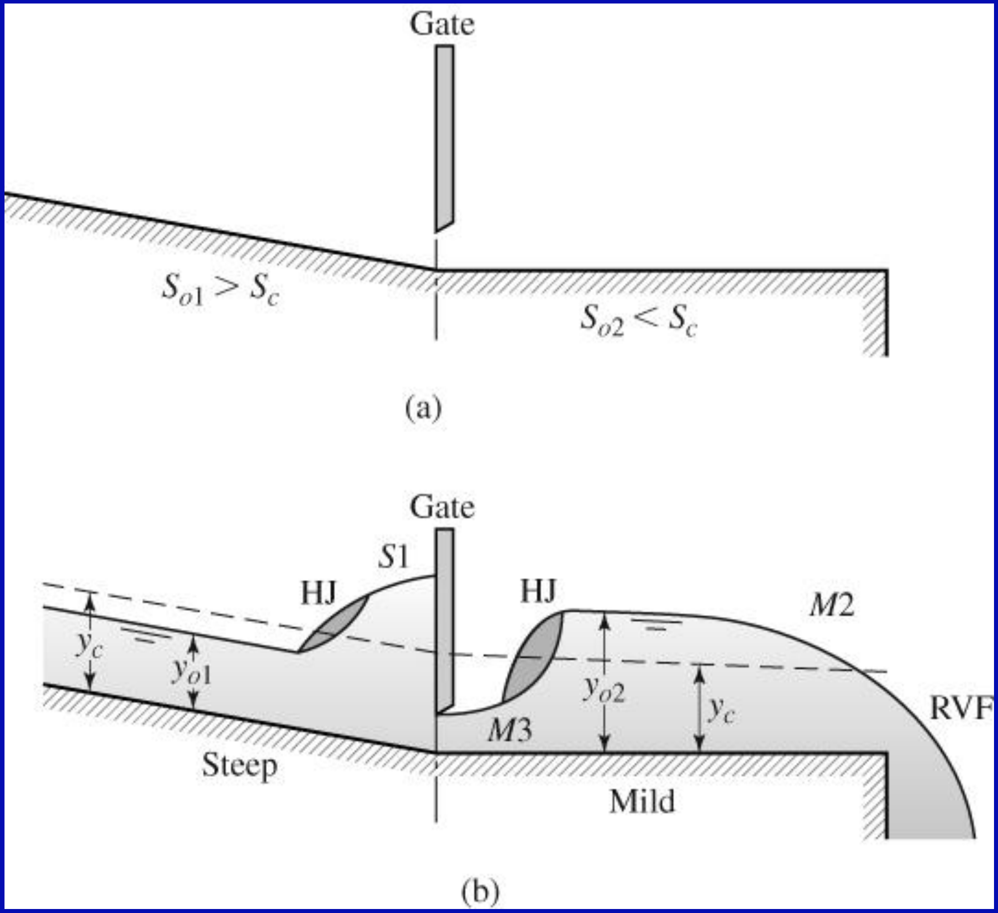
(a) The open channel and gate location.

(b) Critical and normal depths.

(c) Water surface profile.

Example

Draw water surface profile for two reaches of the open channel given in Figure below. A gate is located between the two reaches and the second reach ends with a sudden fall.

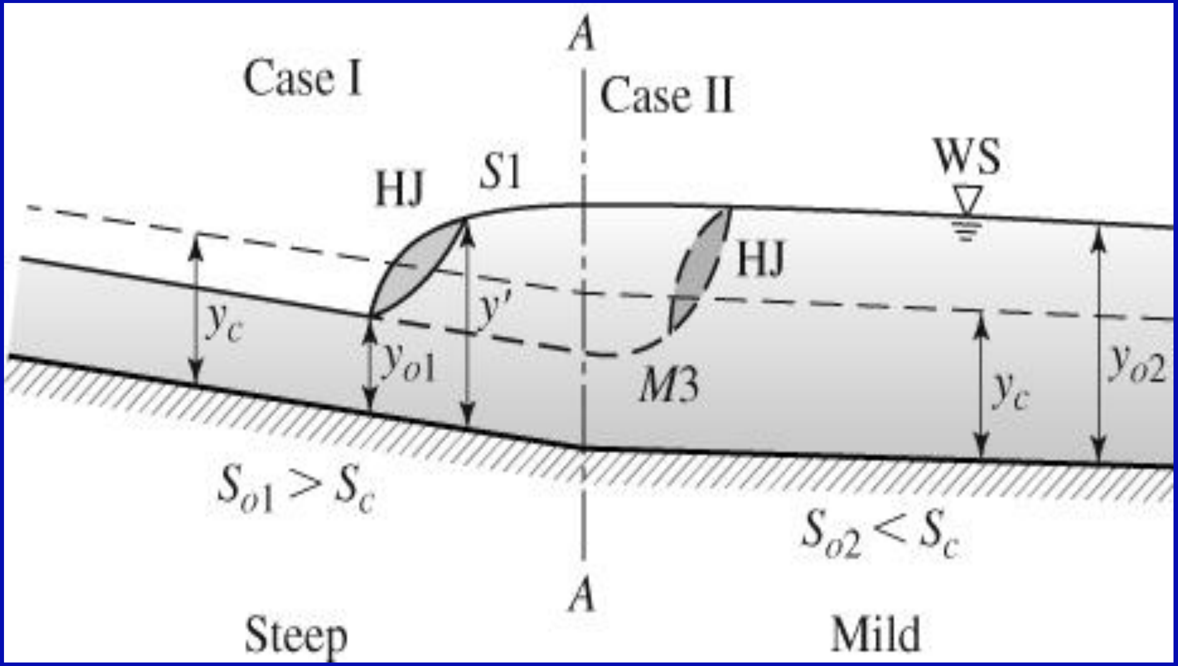


(a) The open channel and gate location.

(b) Water surface profile.

Jump Location and Water Surface Profiles

If hydraulic jump is formed, two different locations are expected for the jump according to the normal depths y_{o1} and y_{o2}.



y_{o1} is known

Calculate conjugate depth of the jump y'

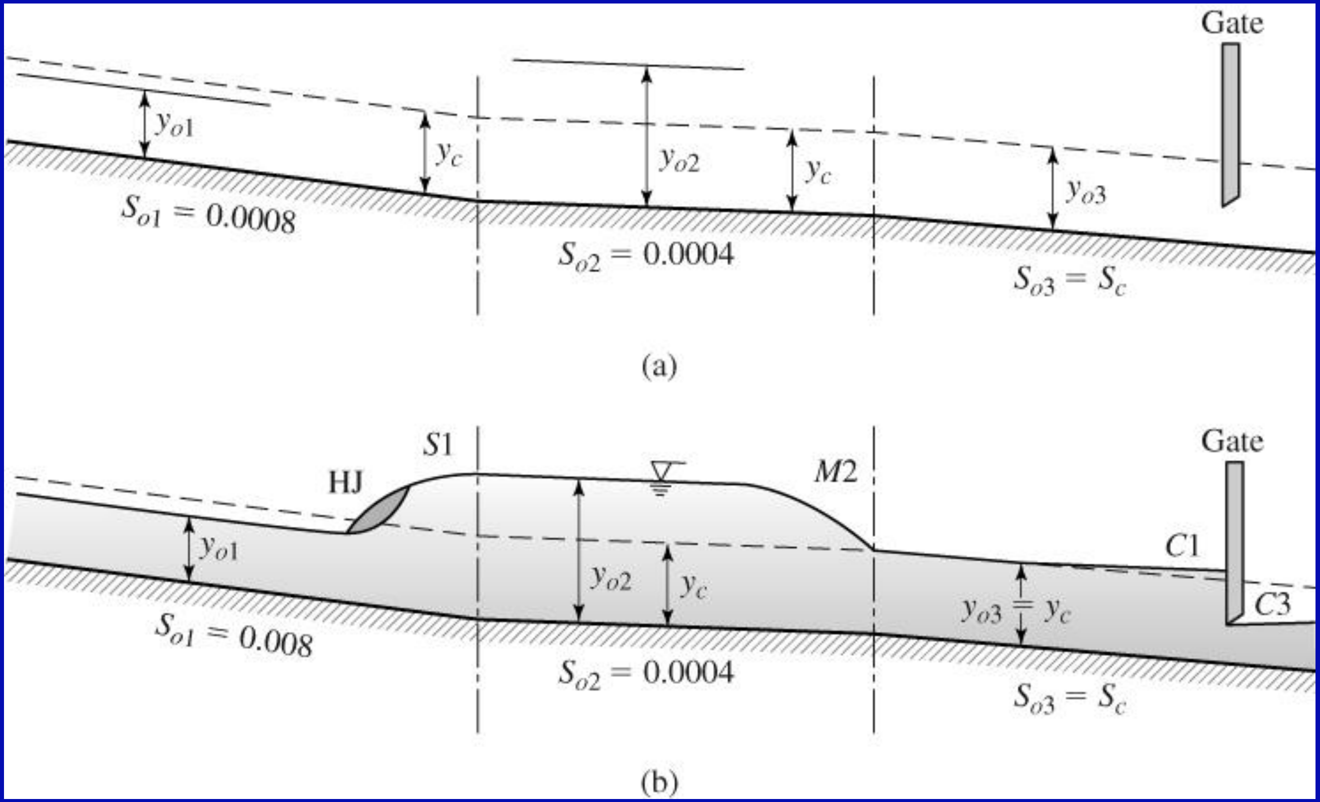
If $y' < y_{o2}$ Case I

If $y' > y_{o2}$ Case II

Example

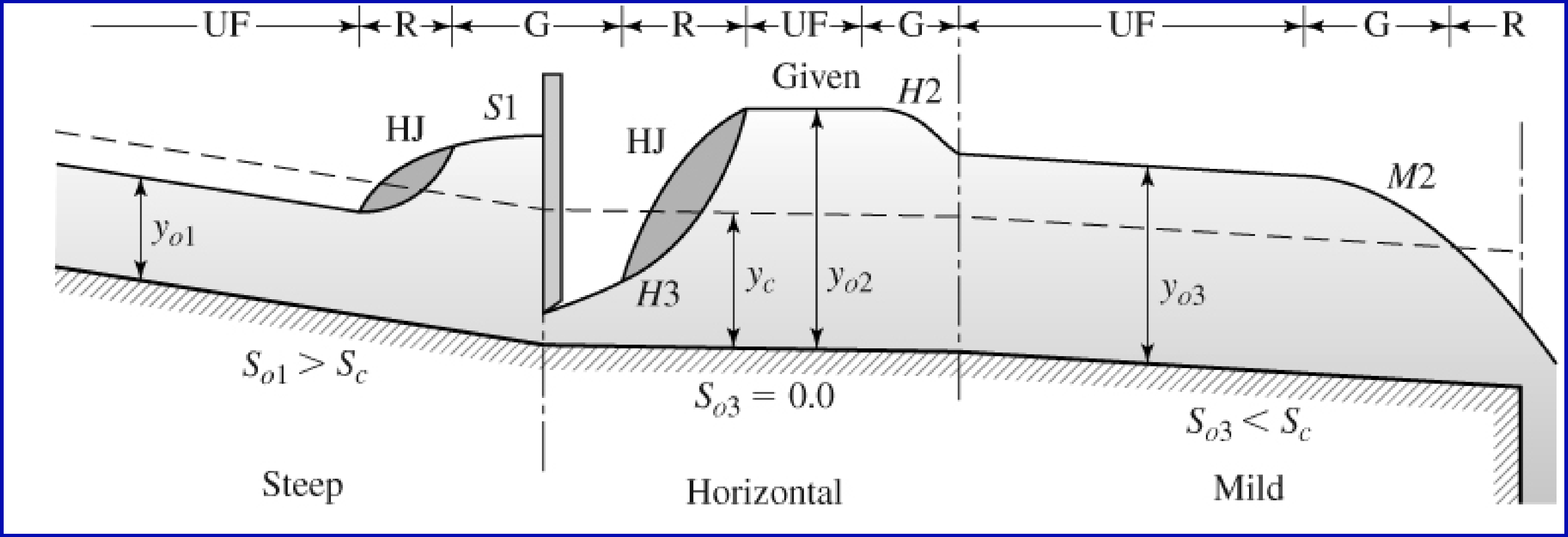
A wide rectangular channel carries a specific discharge of 4.0 m^2/s. The channel consists of three long reaches with bed slope of 0.008, 0.0004 and Sc respectively. A gate located at the end of the last reach. Draw water surface profile. Manning's n=0.016.

First calculate y_c, y_{o1}, y_{o2}, and realize that $y_c = y_{o3}$. To know whether the jump will occur in the first or second reach, calculate y' (subcritical depth) of the jump. If y' < y_{o2} then the jump will take place in the first reach.

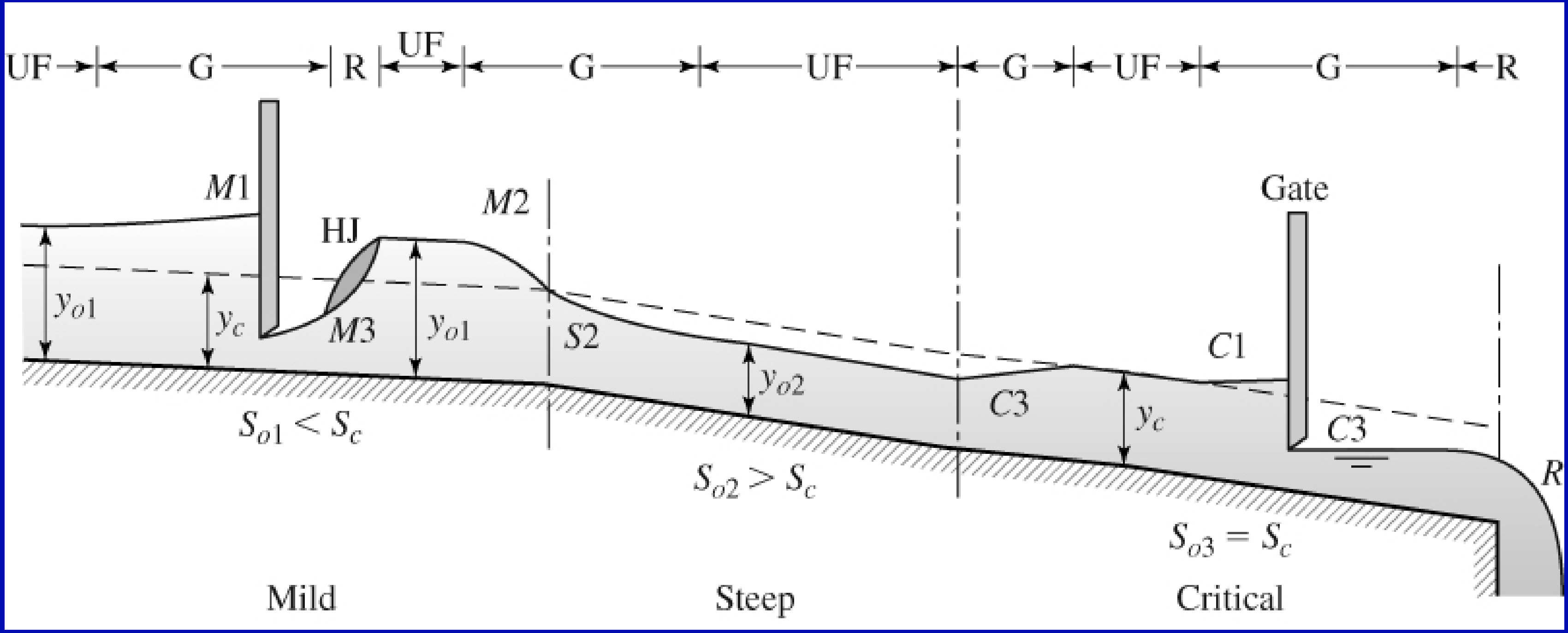


Please see Example 14.10 in your text book.

Example

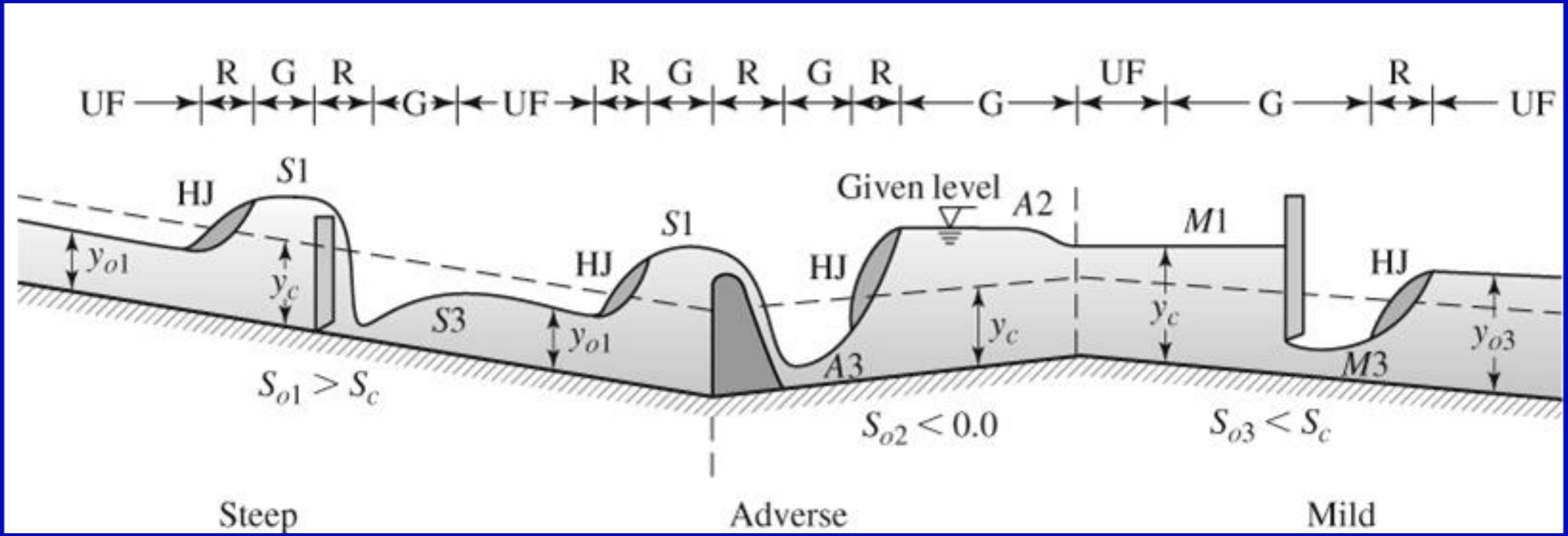


UF
R
G
R
UF
G
UF
G
R
Given
H2
S1
HJ
HJ
M2
y_{o1}
y_c
y_{o2}
y_{o3}
H3
$S_{o1} > S_c$
$S_{o3} = 0.0$
$S_{o3} < S_c$
Steep
Horizontal
Mild

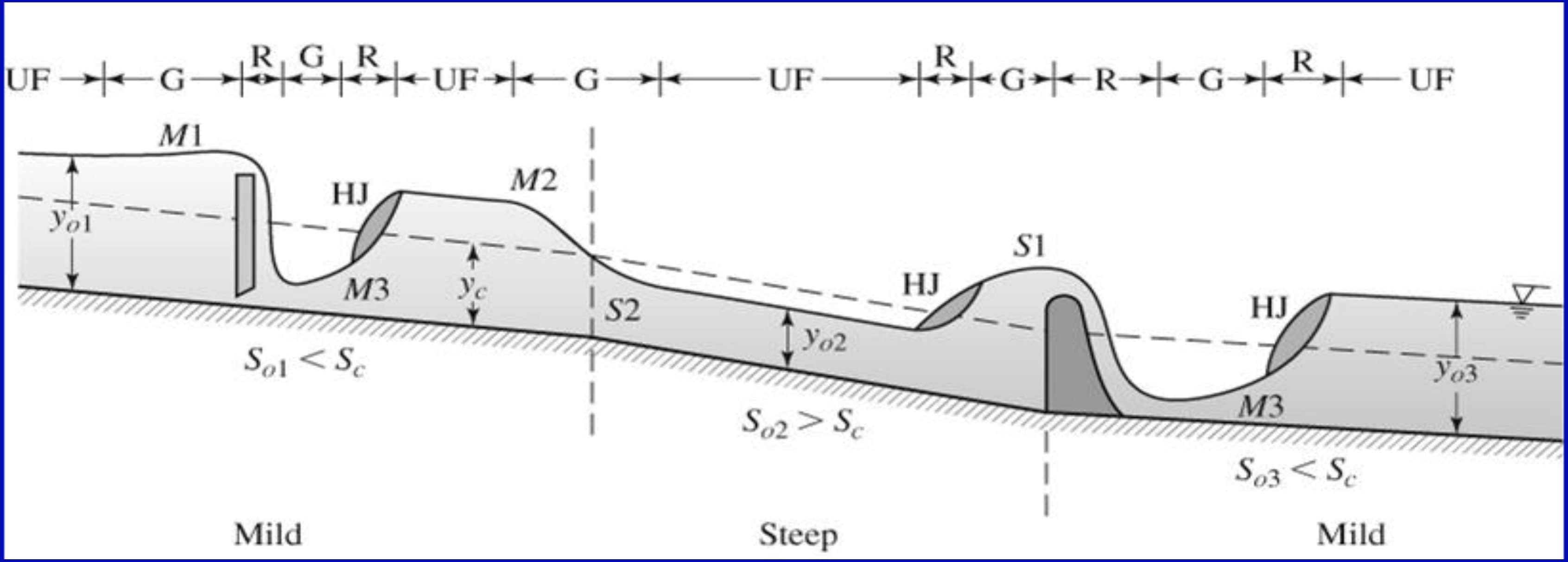


UF
G
R
UF
G
UF
G
UF
G
R
M1
HJ
M2
Gate
y_{o1}
y_c
M3
y_{o1}
S2
y_{o2}
C3
y_c
C1
C3
R
$S_{o1} < S_c$
$S_{o2} > S_c$
$S_{o3} = S_c$
Mild
Steep
Critical

Example



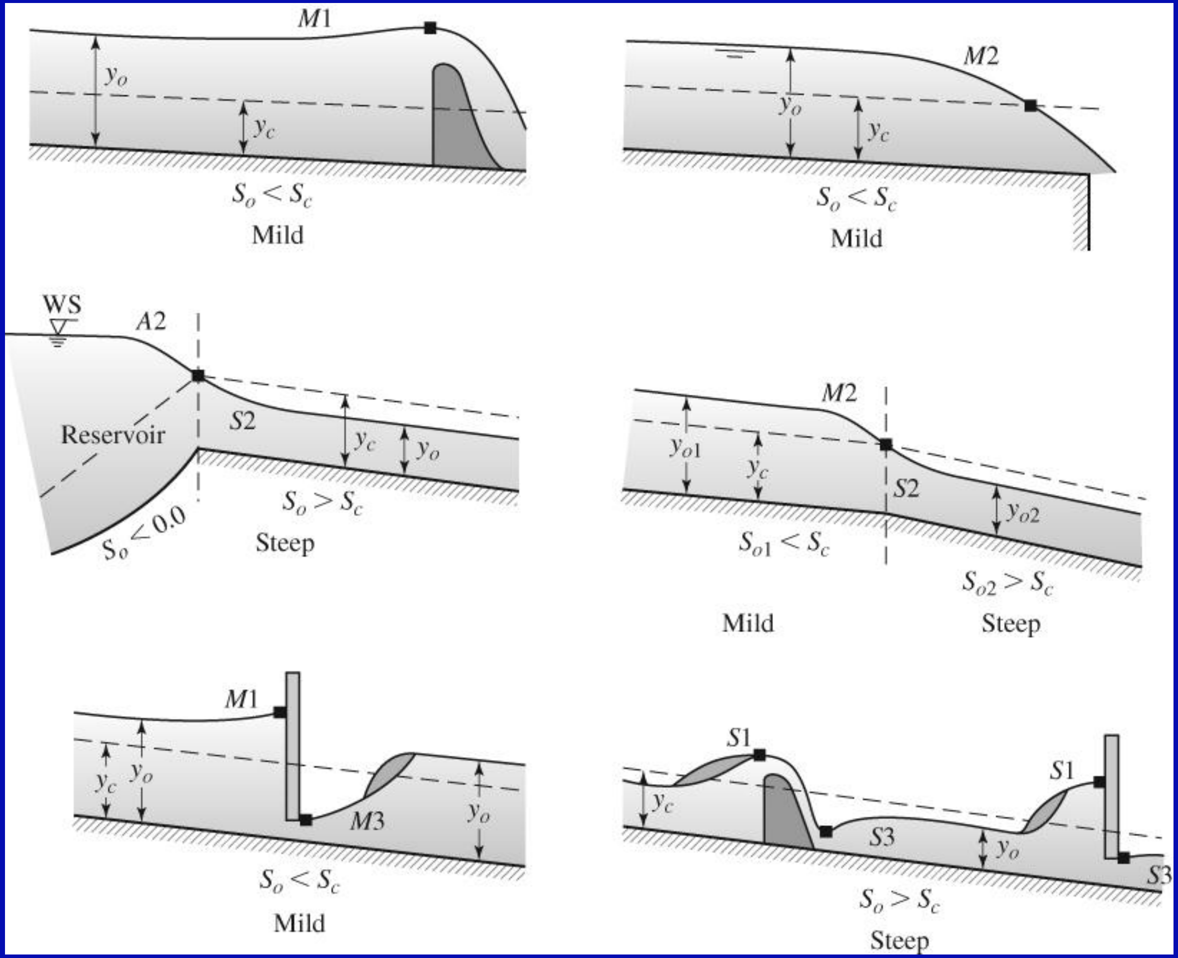
UF
R
G
R
G
UF
R
G
R
G
R
G
UF
G
R
UF
HJ
S1
y_{o1}
y_c
S3
HJ
S1
y_{o1}
$S_{o1} > S_c$
Given level
A2
HJ
y_c
A3
$S_{o2} < 0.0$
M1
y_c
HJ
y_{o3}
M3
$S_{o3} < S_c$
Steep
Adverse
Mild



UF
G
R
G
R
UF
G
UF
R
G
R
G
R
UF
M1
y_{o1}
HJ
M2
M3
y_c
$S_{o1} < S_c$
S2
y_{o2}
$S_{o2} > S_c$
HJ
S1
HJ
y_{o3}
M3
$S_{o3} < S_c$
Mild
Steep
Mild

CONTROL SECTIONS

Bold squares show the control sections.



Control section is a section where a unique relationships between the discharge and the depth of flow.

Gates, weir, and sudden falls and critical depth of are some example of control sections.

Subcritical flows have theirs CS at downstream

Supercritical flows have theirs CS at upstream

Computation
of
Water Surface Profiles

METHODS OF SOLUTIONS OF THE GRADUALLY VARIED FLOW

1. Direct Integration
2. Graphical Integration
3. Numerical Integration

i- The direct step method (distance from depth for regular channels)

ii- The standard step method, regular channels (distance from depth for regular channels)

iii- The standard step method, natural channels (distance from depth for regular channels)

GRADUALLY VARIED FLOW
Important Formulas

$$H = z_b + y + \frac{V^2}{2g}$$

$$E = y + \frac{V^2}{2g}$$

$$H = z_b + E$$

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dE}{dx}$$

$$\frac{dE}{dx} = S_o - S_f$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$

GRADUALLY VARIED FLOW COMPUTATIONS

$$\frac{dE}{\mathrm{dx}} = S_o - \bar{S}_f \qquad \frac{\mathrm{d}\,y}{\mathrm{dx}} = \frac{\mathrm{S}_0 - \mathrm{S}_f}{1 - \mathrm{Fr}^2}$$

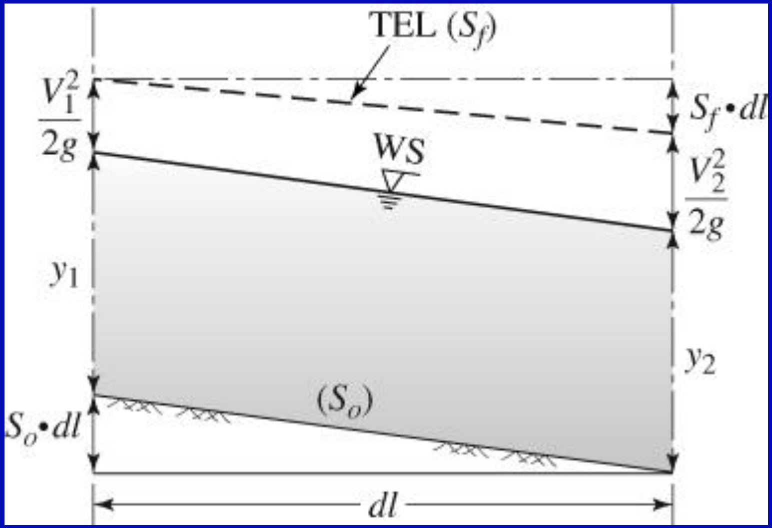
E: specific energy

Analytical solutions to the equations above not available for the most typically encountered open channel flow situations.

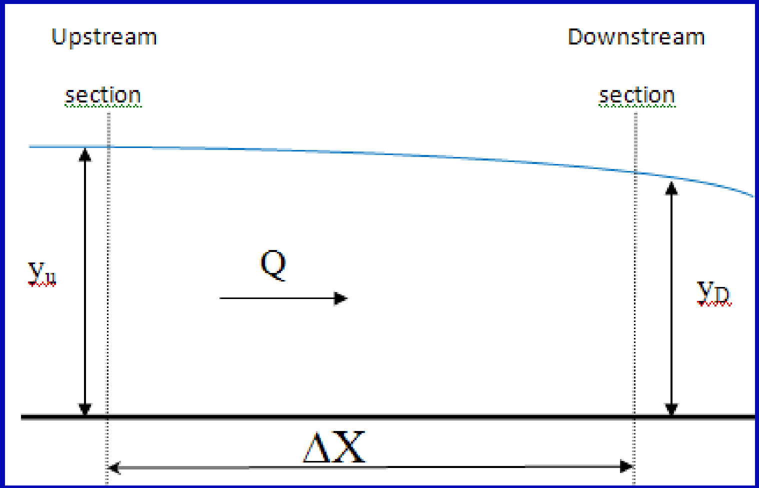
A ***finite difference*** approach is applied to the GVF problems.

Channel is divided into short reaches and computations are carried out from one end of the reach to the other.

DIRECT STEP METHOD



A nonuniform water surface profile



Manning Formula is sufficient to accurately evaluate the slope of total energy line, S_f

$$\frac{E_D - E_U}{\Delta x} = S_o - \bar{S}_f$$

$\bar{S}_f$: average friction slope in the reach

$$\bar{S}_f = \frac{1}{2}(S_{fu} + S_{fD})$$

$$S_{fu} = \frac{n^2 V_u^2}{R_u^{4/3}}$$

$$S_{fD} = \frac{n^2 V_D^2}{R_D^{4/3}}$$

DIRECT STEP METHOD

$$\Delta X = \frac{E_D - E_U}{S_o - \bar{S}_f} = \frac{\left(y_D + V_D^2/2g\right) - \left(y_U + V_U^2/2g\right)}{S_o - \bar{S}_f}$$

Subcritical Flow

The condition at the ***downstream*** is known

y_D, V_D and S_{fD} are known

Chose an appropriate value for y_u

Calculate the corresponding V_u , S_{fu} and $\bar{S}_f$

Then Calculate ΔX

Supercritical Flow

The condition at the ***upstream*** is known

y_u, V_u and S_{fu} are known

Chose an appropriate value for y_D

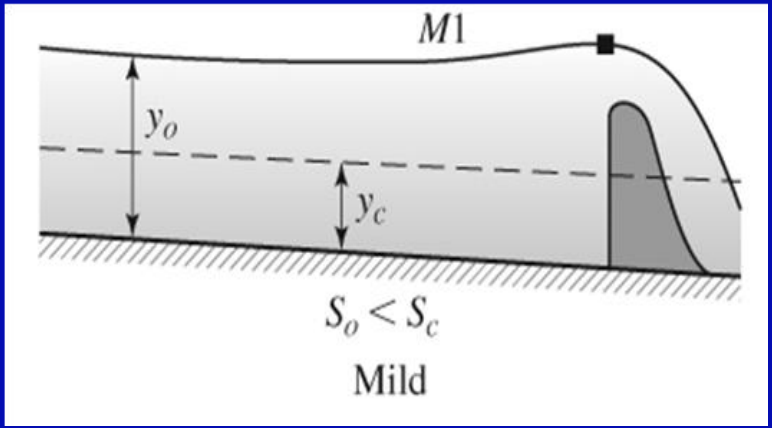
Calculate the corresponding S_{fD}, V_D and $\bar{S}_f$

Then Calculate ΔX

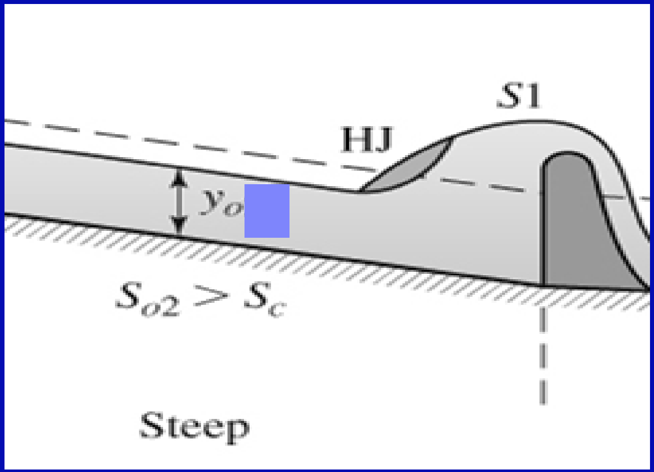
Example

A trapezoidal concrete-lined channel has a constant bed slope of 0.0015, a bed width of 3 m and side slopes 1:1. A control gate increased the depth immediately upstream to **4.0m** when the discharge is 19 m^3/s. Compute WSP to a depth 5% greater than the uniform flow depth (n=0.017).

Two possibilities exist:



OR



Solution

The first task is to calculate the critical and normal depths.

Using Manning formula, the depth of uniform flow:

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

y_o = 1.75 m

Using the critical flow condition, the critical depth:

$$Fr^2 = \frac{Q^2 T}{g A^3}$$

y_c = 1.36 m

It can be realized that the profile should be **M1** since **y_o > yc**
That is to say, the possibility is valid in our problem.

Solution

$$\Delta X = \frac{E_D - E_U}{S_o - \bar{S}_f} = \frac{\left(y_D + V_D^2/2g\right) - \left(y_U + V_U^2/2g\right)}{S_o - \bar{S}_f}$$

y	A	R	E	ΔE	S_f	$\bar{S}_f$	So-S_f	Δx	x
4.000	28.00	1.956	4.023		0.000054				0
3.900	26.91	1.918	3.925	0.098	0.000060	0.000057	0.001443	67.98	67.98
3.800	25.84	1.880	3.828	0.098	0.000067	0.000064	0.001436	68.14	136.11
3.700	24.79	1.841	3.730	0.098	0.000075	0.000071	0.001429	68.32	204.44
3.600	23.76	1.802	3.633	0.097	0.000084	0.000080	0.001420	68.54	272.98
3.500	22.75	1.764	3.536	0.097	0.000095	0.000089	0.001411	68.80	341.78
3.400	21.76	1.725	3.439	0.097	0.000107	0.000101	0.001399	69.09	410.87
3.300	20.79	1.686	3.343	0.096	0.000120	0.000113	0.001387	69.44	480.31
3.200	19.84	1.646	3.247	0.096	0.000136	0.000128	0.001372	69.86	550.17
3.100	18.91	1.607	3.151	0.095	0.000155	0.000146	0.001354	70.36	620.53
3.000	18.00	1.567	3.057	0.095	0.000177	0.000166	0.001334	70.96	691.49
1.800	8.64	1.068	2.047	0.066	0.001280	0.001163	0.000337	195.10	1840.24

$y_o + (0.05\ y_o)$

THE STANDART STEP METHOD

- Applicable to non-prismatic channels and therefore to natural river

- Objectives

- To calculate the surface elevations at the station with predetermined the station positions

- A trial and error method is employed

THE STANDART STEP METHOD

$$\frac{\Delta E}{dx} = S_o - S_f$$

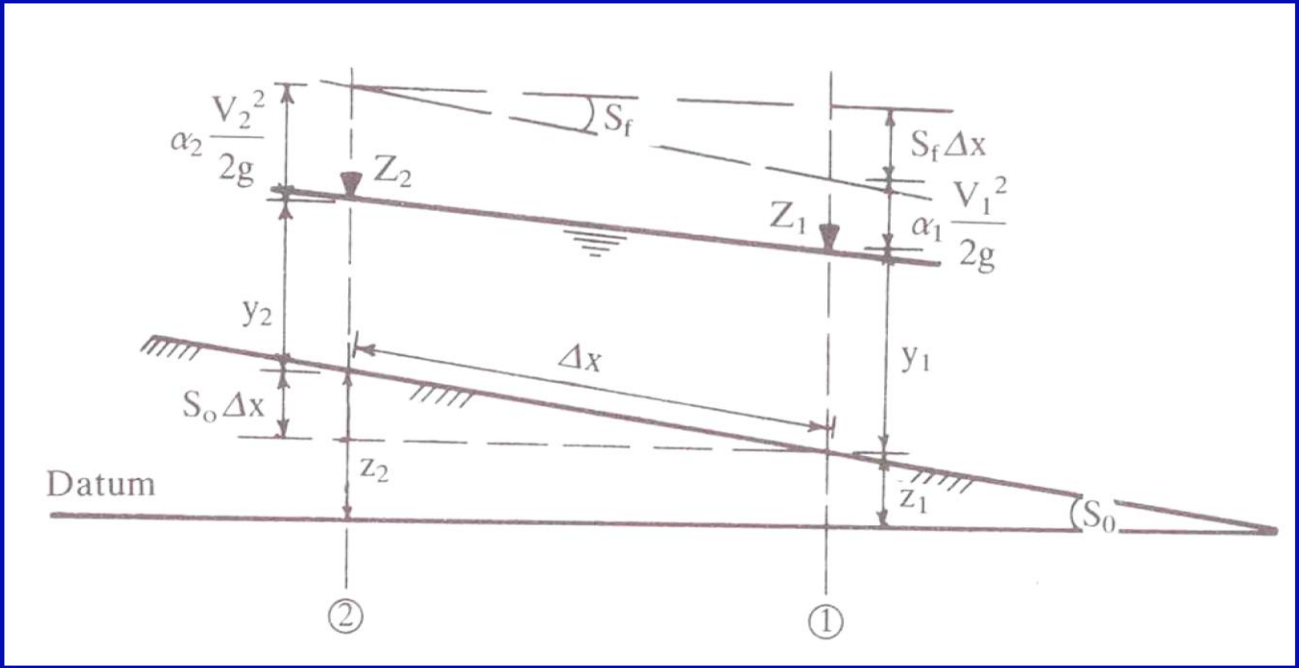
This can be rewritten in finite difference form

$$\Delta E_s = \Delta X(S_o - \bar{S}_f)_{mean}$$

where ‘mean’ refers to the average values for the interval ΔX.

This form of the equation may be used to determine the depth given distance intervals. The solution method is an iterative procedure as follows;

THE STANDART STEP METHOD



$$y_1 + \alpha \frac{V_1^2}{2g} + h_f = S_o \Delta X + y_2 + \alpha \frac{V_2^2}{2g}$$

$$Z_1 = y_1 \qquad Z_2 = y_2 + S_o \Delta X$$

$$Z_1 + \alpha \frac{V_1^2}{2g} + h_f = Z_2 + \alpha \frac{V_2^2}{2g}$$

$$H_1 = Z_1 + \alpha \frac{V_1^2}{2g}; \qquad H_2 = Z_2 + \alpha \frac{V_2^2}{2g}$$

$$H_1 = h_f + H_2$$

THE STANDART STEP METHOD

H_1 is known and ΔX predetermined.

1) Assume a value for depth (Z_2); simple add a small amount to Z_1
2) Calculate y_2 from $y_2 = Z_2 - So\Delta X$
3) Calculate the corresponding specific energy (E_2)
4) Calculate the corresponding friction slope S_2
5) Calculate H_2
6) Calculate $H_1 = H_2 + S_f \Delta X$
7) Compare H_2 and H_1 if the differences is not within the prescribed limit (e.g., 0.001m) re-estimate Z_2 and repeat the procedure until the agreement is reached.

THE STANDART STEP METHOD

X (m)	Z (m)	y (m)	A (m2)	V (m/s)	aV2/(2g)	H (1)	R (m)	Sf	$\overline{S}_f$	Δx	hf	H (2)
0	4	4	28	0.679	0.026	4.026	1.956	5E-05	-			
100	4.003	3.853	26.405	0.72	0.029	4.032	1.9	6E-05	6E-05	100	0.0059	4.032
200	4.005	3.705	24.842	0.765	0.033	4.039	1.843	7E-05	7E-05	100	0.0069	4.038
300	4.009	3.559	23.343	0.814	0.037	4.047	1.786	9E-05	8E-05	100	0.0082	4.046
400	4.015	3.415	21.907	0.867	0.042	4.057	1.731	0.0001	1E-04	100	0.0096	4.057
500	4.02	3.27	20.503	0.927	0.048	4.068	1.674	0.0001	0.0001	100	0.0115	4.068
1300	4.153	2.203	11.462	1.658	0.154	4.307	1.242	0.0006	0.0005	100	0.0541	4.307
1400	4.195	2.095	10.674	1.78	0.178	4.373	1.196	0.0007	0.0007	100	0.0658	4.373
1500	4.25	2	10	1.9	0.202	4.452	1.155	0.0009	0.0008	100	0.0791	4.452
1600	4.318	1.918	9.433	2.014	0.227	4.545	1.12	0.001	0.0009	100	0.0934	4.545
1700	4.402	1.852	8.986	2.114	0.251	4.653	1.091	0.0012	0.0011	100	0.1079	4.653
1800	4.505	1.805	8.673	2.191	0.269	4.774	1.07	0.0013	0.0012	100	0.1209	4.774
1900	4.621	1.771	8.449	2.249	0.284	4.905	1.055	0.0014	0.0013	100	0.1314	4.905